Kindergarten

2018-2019 Guide

February 11th – April 18th

Eureka

Module 4: *Number Pairs, Addition and Subtraction to 10*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 4 Performance Overview

- In Topic A, decompositions and compositions of numbers to 5 are revisited to reinforce how a whole can be broken into two parts and how two parts can be joined to make a whole. Decomposition and composition are taught simultaneously using the number bond model so students begin to understand the relationship between parts and wholes before adding and subtracting.

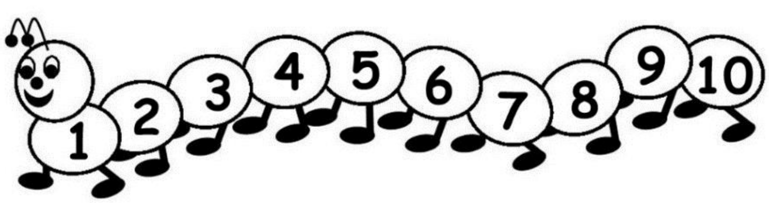
- Topic B continues with decomposing and composing 6, 7, and 8 using the number bond model. Students systematically work with each quantity, finding all possible number pairs using story situations, objects, sets, arrays, 5 + n patterns, 1 and numerals.

- Topic C introduces addition to totals of 6, 7, and 8 within concrete and pictorial settings, first generating number sentences without unknowns (e.g., 5 + 2 = 7) to develop an understanding of the addition symbol and the referent of each number within the equation. Next, students graduate to working within the addition word problem types taught in kindergarten.

- Topic D introduces subtraction with 6, 7, and 8 with no unknown which build from the concrete level of students acting out, crossing out objects in a set, and breaking and hiding parts, to more formal representations of decomposition recorded as or matched to equations

- Topics E, F, and G parallel the first half of the module with the numbers 9 and 10. Topic E explores composition, decomposition, and number pairs using the number bond model. It is essential that students build deep understanding and skill with identifying the number pairs of 6 through 10. Topics F and G deal with addition and subtraction, respectively. Students are refocused on representing larger numbers by drawing the 5 + n pattern to bridge efficiently from seeing the embedded five to representing that as addition.

- Topic H explores the behavior of zero: the additive identity. Students learn that adding or subtracting zero does not change the original quantity.



Module 4: Numbers Pairs, Addition and Subtraction to 10 ***Pacing:*** February 11th-April 18th 2019 **47 Days**		
Topic	**Lesson**	**Lesson Objective:**
Topic A: Compositions and Decompositions of 2,3,4 and 5	Lesson 1	Model composition and decomposition of numbers to 5 using actions, objects, and drawings.
	Lesson 2	Model composition and decomposition of numbers to 5 using fingers and linking cube sticks
	Lesson 3	Represent composition story situations with drawings using numeric number bonds.
	Lesson 4	Represent decomposition story situations with drawings using numeric number bonds
	Lesson 5	Represent composition and decomposition of numbers to 5 using pictorial and numeric number bonds.
	Lesson 6	Represent number bonds with composition and decomposition story situations
Topic B: Decompositions of 6,7 and 8 into Number Pairs	Lesson 7	Model decompositions of 6 using a story situation, objects, and number bonds
	Lesson 8	Model decompositions of 7 using a story situation, sets, and number bonds.
	Lesson 9	Model decompositions of 8 using a story situation, arrays, and number bonds.
	Lesson 10	Lesson 10: Model decompositions of 6–8 using linking cube sticks to see patterns
	Lesson 11	Lesson 11: Represent decompositions for 6–8 using horizontal and vertical number bonds
	Lesson 12	Use 5-groups to represent the 5 + n pattern to 8.
Topic C: Addition with Totals of 6,7 and 8	Lesson 13	Represent decomposition and composition addition stories to 6 with drawings and equations with no unknown
	Lesson 14	Represent decomposition and composition addition stories to 7 with drawings and equations with no unknown.
	Lesson 15	Represent decomposition and composition addition stories to 8 with drawings and equations with no unknown.
	Lesson 16	Solve add to with result unknown word problems to 8 with equations. Box the unknown.

	Lesson 17	Solve put together with total unknown word problems to 8 using objects and drawings.
	Lesson 18	Solve both addends unknown word problems to 8 to find addition patterns in number pairs.
Topic D: Subtraction from Numbers to 8	Lesson 19	Use objects and drawings to find how many are left.
	Lesson 20	Solve take from with result unknown expressions and equations using the minus sign with no unknown.
	Lesson 21	Represent subtraction story problems using objects, drawings, expressions, and equations.
	Lesson 22	Decompose the number 6 using 5-group drawings by breaking off or removing a part and record each decomposition with a drawing and subtraction equation.
	Lesson 23	Decompose the number 7 using 5-group drawings by hiding a part and record each decomposition with a drawing and subtraction equation.
	Lesson 24	Decompose the number 8 using 5-group drawings and crossing off a part, and record each decomposition with a drawing and subtraction equation.
Mid- Module Assessment (Interview Style: 3 days) March 22-26th 2019		
Topic E: Decomposition of 9 and 10 into Number Pairs	Lesson 25 & 26	Model decompositions of 9 using a story situation, objects (linking cubes, fingers) and number bonds.
	Lesson 27 & 28	Model decompositions of 10 using a story situation, objects (linking cubes, fingers) sets and number bonds
Topic F: Addition with totals 9 and 10	Lesson 29 & 30	Represent pictorial decomposition and composition addition stories to 9 and 10 with 5-group drawings and equations with no unknown.
	Lesson 31	Solve add to with total unknown and put together with total unknown problems with totals of 9 and 10.
	Lesson 32	Solve both addends unknown word problems with totals of 9 and 10 using 5-group drawings
Topic G: Subtraction from 9 and 10	Lesson 33 &34	Solve take from equations with no unknowns using numbers to 10. Represent subtraction story problems by breaking off, crossing out, and hiding a part.
	Lesson 35 & 36	Decompose the number 9 and 10 using 5-group drawings, and record each decomposition with a subtraction equation.
Topic H:	Lesson 37	Add or subtract 0 to get the same number and relate to word problems wherein the same quantity that joins a set, separates.

Patterns with Adding 0 and 1 and Making 10	Lesson 38	Add 1 to numbers 1–9 to see the pattern of the next number using 5-group drawings and equations.
	Lesson 39 & 40	Find the number that makes 10 for numbers 1–9, and record each with a 5-group drawing and an addition equation
	Lesson 41	Culminating task—choose tools strategically to model and represent a stick of 10 cubes broken into two parts.
End-of- Module Assessment (Interview Style: 3 days) **April 16-18th 2019**		

NJSLS Standards:

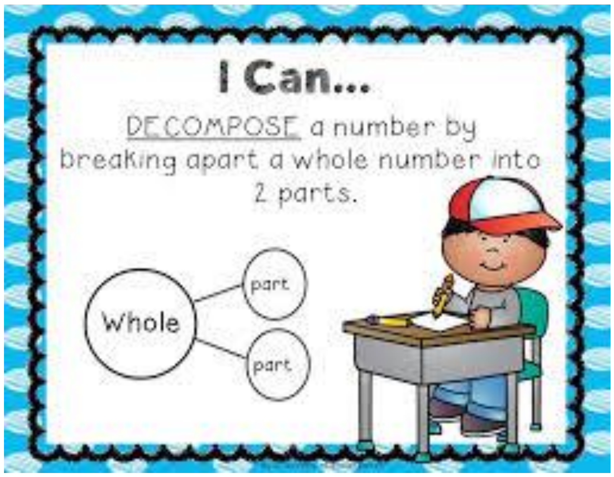
Module 4: Number Pairs, Addition and Subtraction to 10	
K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawing, sounds (e.g claps) acting out situations, verbal explanations, expressions, or equations. (drawings need not show details, but should show the mathematics in the problem)
• Students develop an understanding of the meaning of addition and subtraction by modeling how they can put together (compose) or take apart (decompose) up to 10 objects in different ways • It is critical for students to have a variety of experiences with concrete materials, progress to drawing pictures to express their thinking, and finally see written addition and subtraction expression and equations. • Teachers may write equations if students are not ready to do this on their own • Pose questions that ask students to explain their work using pictures and words • Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.” Please note that it is not until First Grade when “Understand the meaning of the equal sign” is an expectation	
K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g by using objects or drawings to represent the problem
• Limit problems to the situations identified for kindergarten in Table 1 • Pose questions that have students explain their thinking and make connections to their previous work with addition and subtraction meanings (K.OA.A.1) • It is critical that students connect what to do with the actions or problem • situation and use models rather than to look for clue words • Students should have multiple experiences with each situation below but do not need to identify the situation by name **Example:** Add to-result unknown: Two bunnies sat on the grass.Three more bunnies hopped there. How many bunnies are on the grass now? 2+3=? **Example:** Take from- result unknown: Five apples were on the table.I ate two apples. How many apples are on the table now? 5–2=?	

Example: Put together/take apart- total unknown: Three red apples and two green apples are on the table.How many apples are on the table? 3+2=?

Example: Put together/take apart- addend unknown: Five apples are on the table.Three are red and the rest are green.How many apples are green? 3+?=5,5–3=?

K.OA.A.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5=2+3 and 5=4+1)



- Students develop an understanding of part-whole relationships as they recognize that a set of objects (5) can be broken into smaller sub-sets (3 and 2) and still remain the total amount (5). In addition, this objective asks students to realize that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decompose), students use the understanding that a smaller set of objects exists within that larger set (inclusion)

- In Kindergarten, students need ample experiences breaking apart numbers and using the vocabulary "and" & "same amount as" before symbols (+, =) and equations (5= 3 + 2) are introduced. If equations are used, a mathematical representation (picture, objects) needs to be present as well. **Example**: "Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas. Students could draw pictures of: 4 blue and 1 red button 3 blue and 2 red buttons 2 blue and 3 red buttons 1 blue and 4 red buttons

- Although it is appropriate for kindergarteners to use their fingers in initial counting and exploration experiences, focus on concrete and pictorial representations to develop an understanding that numbers can be put together and taken apart in a variety of ways.

K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation

- Students build upon the understanding that a number (less than or equal to 10) can be decomposed into parts (K.OA.3) to find a missing part of 10. Through numerous concrete experiences, kindergarteners model the various sub-parts of ten and find the missing part of 10.
  Example: When working with 2-color beans, a student determines that 4 more beans are needed to make a total of 10.

- In addition, kindergarteners use various materials to solve tasks that involve decomposing and composing 10.
  Example: Example: "A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?
  Student A: Using a Ten-Frame "I used a ten frame for the case. Then, I put on 6 counters for juice still in the case. There's no juice in these 4 spaces. So, 4 are missing."

  Student B: Think Addition "I counted out 10 counters because I knew there needed to be ten. I pushed these 6 over here because they were in the container. These are left over. So there's 4 missing."

  Student C: Fluently add/subtract "I know that it's 4 because 6 and 4 is the same amount as 10."

- Children's literature can offer many contexts for problems.

- Ten frames and rekenreks are very powerful visual aids for this standard

K.OA.A 5	Fluently add and subtract within 5

- Students are fluent when they display accuracy (correct answer), efficiency (a reasonable amount of steps in about 3 seconds without resorting to counting), and flexibility (using strategies such as the distributive property). Students develop fluency by understanding and internalizing the relationships that exist between and among numbers.

- In order to fluently add and subtract, children must first be able to see sub-parts within a number (Hierarchical inclusion, K.CC.4.c).

- Children need repeated experiences with many different types of concrete materials (such as cubes, chips, and buttons) over an extended amount of time in order to recognize that there are only particular sub-parts for each number. Therefore, children will realize that if 3 and 2 is a combination of 5, then 3 and 2 cannot be a combination of 6.

- Traditional flash cards or timed tests have not been proven as effective instruc-

tional strategies for developing fluency. Rather, numerous experiences with breaking apart actual sets of objects and developing relationships between numbers help children internalize parts of number and develop efficient strategies for retrieval.
M : Major Content S: Supporting Content A : Additional Content

Teaching Representations/ Manipulatives/ Tools:	
• Hula hoops • Linking cubes • Number bonds • Number path • Number towers • Ten frames	• Showing fingers the Math way • Sets of objects • 5-group dot cards • Rekenrek (Slavonic abacus having beads with a color change at the five)

Module 4 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math *Module 4: Number Pairs, Addition and Subtraction to 10*			
Optional Mid-Module Assessment (Interview Style)	K.OA.1 K.OA.2 K.OA.3 K.OA.5	1 Block	Individual or Small Group with Teacher
Optional End of Module Assessment (Interview Style)	K.OA.1 K.OA.2 K.OA.3 K.OA.4	1 Block	Individual or Small Group with Teacher

Kindergarten Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Independent
Exit Ticket

50-60 min.

CENTERS/ LEARNING STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

15-20 min.

M:	A:	T:	H:
Meet with the teacher	Application/ Problem Solving	Technology	Hands on Activities

SUMMARY: Whole Group
Lesson Closure: Student Reflection; Real Life Connections to Concept

Up to 5 min.

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Number Talks Cheat Sheet

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?
- What are Number Talks and Why are they

Terminology

- Addition (specifically using add to with result unknown, put together with total unknown, put together with both addends unknown)
- Addition and Subtraction sentences (equations)
- Make 10 (combine two numbers from 1–9 that add up to 10)
- Minus (–)
- Number bond (mathematical model)
- Number pairs or partners (embedded numbers)
- Part (addend or embedded number)
- Put together (add)
- Subtraction (specifically using take from with result unknown)
- Take apart (decompose)
- Take away (subtract)
- Whole (total)

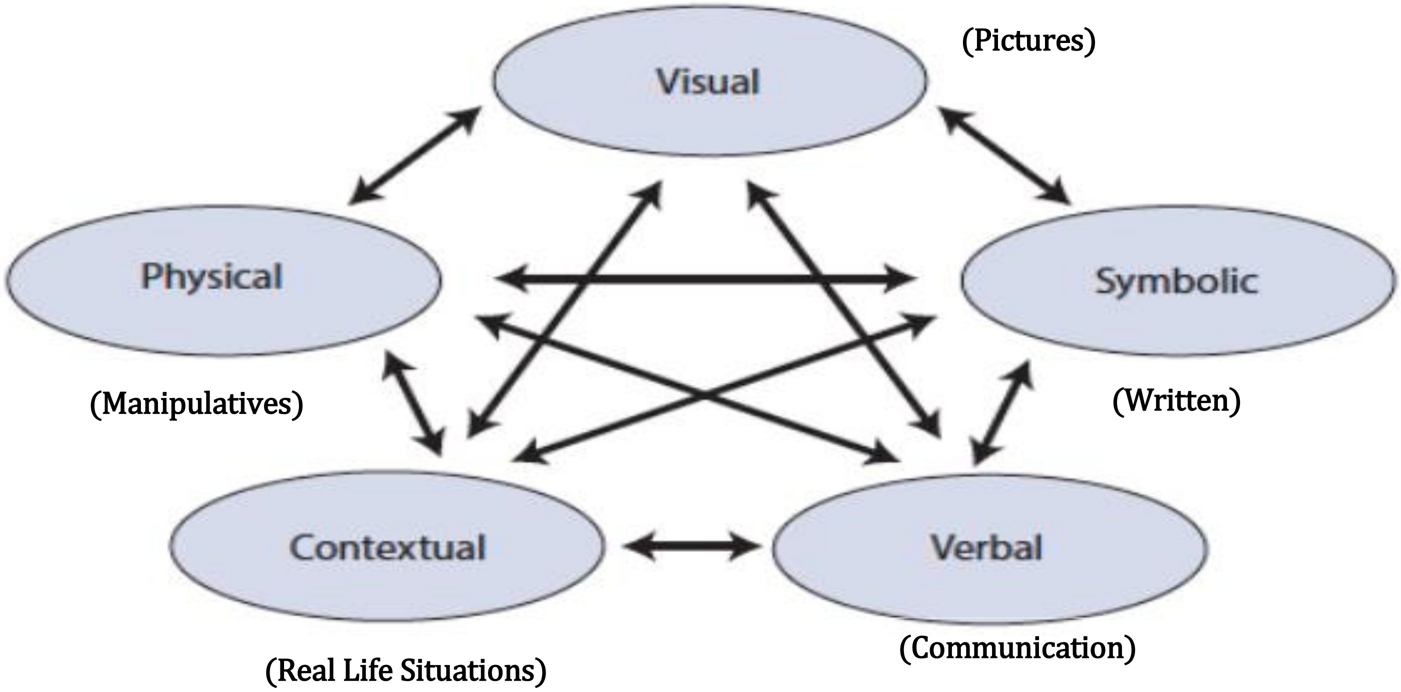
PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	Math Practices
K.OA.A.5	Fluently add and subtract within 5 Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	i) Tasks should provide students opportunities to demonstrate fluency for addition and subtraction within 5 and to apply different solution methods. ii) Interviews (individual or small group) should target students' abilities to meet this evidence statement.	MP.5 MP.7.

Student Name: ______________________ Task: ________________ School: ______ Teacher: ________ Date: ______

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

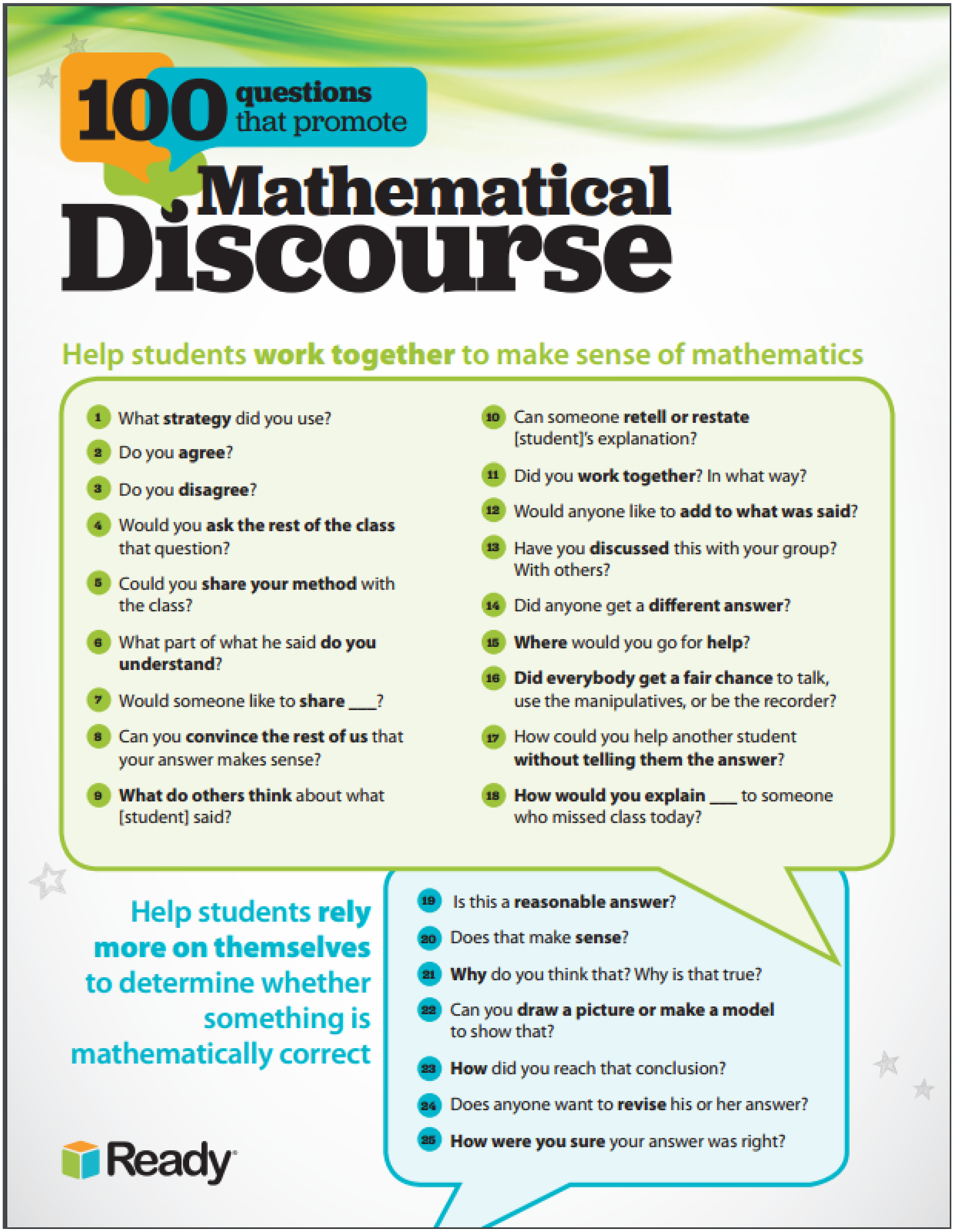
Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

<u>**Teacher Questioning:**</u>
Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts.** What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation
K.OA.5 Add and Subtract within 5.
1.OA.6 Add and Subtract within 10.
2.OA.2 Add and Subtract within 20**.**

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

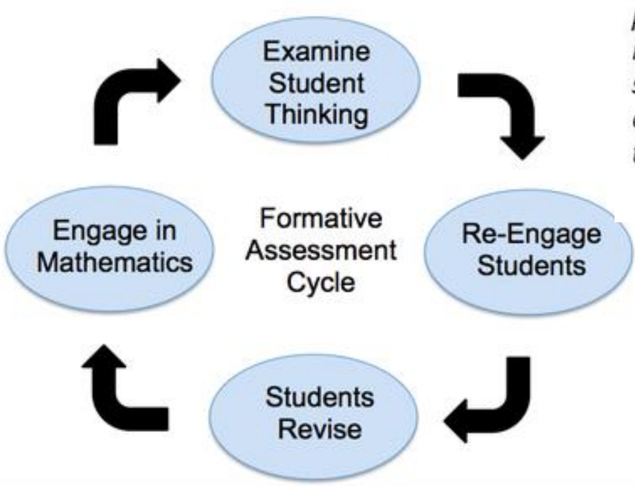
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.	
1	**Make sense of problems and persevere in solving them** In Kindergarten, students learn that doing math involves solving problems and discussing how they solved them. Students will begin to explain the meaning of a problem, and look for ways to solve it. Kindergarteners will learn how to use objects and pictures to help them understand and solve problems. They will begin to check their thinking when the teacher asks them how they got their answer, and if the answer makes sense. When working in small groups or with a partner they will listen to the strategies of the group and will try different approaches.
2	**Reason abstractly and quantitatively** Mathematically proficient students in Kindergarten make sense of quantities and the relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Kindergarten, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 7 children on the playground and some children go line up. If there are 4 children still playing, how many children lined up?" Kindergarten students are expected to translate that situation into the equation: 7-4 = ___, and then solve the task. Students also contextualize situations during the problem solving process. For example, while solving the task above, students refer to the context of the task to determine that they need to subtract 4 since the number of children on the playground is the total number of students except for the 4 that are still playing. Abstract reasoning also occurs when students measure and compare the lengths of objects.

3	**Construct viable arguments and critique the reasoning of others** Mathematically proficient students in Kindergarten accurately use mathematical terms to construct arguments and engage in discussions about problem solving strategies. For example, while solving the task, "There are 8 books on the shelf. If you take some books off the shelf and there are now 3 left, how many books did you take off the shelf?" students will solve the task, and then be able to construct an accurate argument about why they subtracted 3 form 8 rather than adding 8 and 3. Further, Kindergarten students are expected to examine a variety of problem solving strategies and begin to recognize the reasonableness of them, as well as similarities and differences among them.
4	**Model with mathematics** Mathematically proficient students in Kindergarten model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Kindergarten students rely on concrete manipulatives and pictorial representations while solving tasks, but the expectation is that they will also write an equation to model problem situations. For example, while solving the task "there are 7 bananas on the counter. If you eat 3 bananas, how many are left?" Kindergarten students are expected to write the equation $7-3 = 4$. Likewise, Kindergarten students are expected to create an appropriate problem situation from an equation. For example, students are expected to orally tell a story problem for the equation $4+5 = 9$.
5	**Use appropriate tools strategically** Mathematically proficient students in Kindergarten have access to and use tools appropriately. These tools may include counters, place value (base ten) blocks, hundreds number boards, number lines, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students should also have experiences with educational technologies, such as calculators, virtual manipulatives, and mathematical games that support conceptual understanding. During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while solving the task "There are 4 dogs in the park. If 3 more dogs show up, how many dogs are they?" Kindergarten students are expected to explain why they used specific mathematical tools."

6	**Attend to precision**
	Mathematically proficient students in Kindergarten are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Kindergarten describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.
7	**Look for and make use of structure**
	Mathematically proficient students in Kindergarten carefully look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property, in that 1+4 = 5, and 4+1 = 5. While decomposing teen numbers, students realize that every number between 11 and 19, can be decomposed into 10 and some leftovers, such as 12 = 10+2, 13 = 10+3, etc. Further, Kindergarten students make use of structures of mathematical tasks when they begin to work with subtraction as missing addend problems, such as 5- 1 = __ can be written as 1+ __ = 5 and can be thought of as how much more do I need to add to 1 to get to 5?
8	**Look for and express regularity in repeated reasoning**
	Mathematically proficient students in Kindergarten begin to look for regularity in problem structures when solving mathematical tasks. Likewise, students begin composing and decomposing numbers in different ways. For example, in the task "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" Kindergarten students are expected to realize that the 8 crayons could include 4 of each color (4+4 = 8), 5 of one color and 3 of another (5+3 = 8), etc. For each solution, students repeated engage in the process of finding two numbers that can be joined to equal 8.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion. Selection of children is guided by the mathematical goal for the lesson
4. Sequencing	What order will the solutions be shared with the class? Sequence depends largely on the teacher's goals for a lesson
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ___________

NJSLS.:

____
______
____
____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Kindergarten PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	**PLD Level 4: 89% Strong Command**	**PLD Level 3: 79% Moderate Command**	**PLD Level 2: 69% Partial Command**	**PLD Level 1: 59% Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

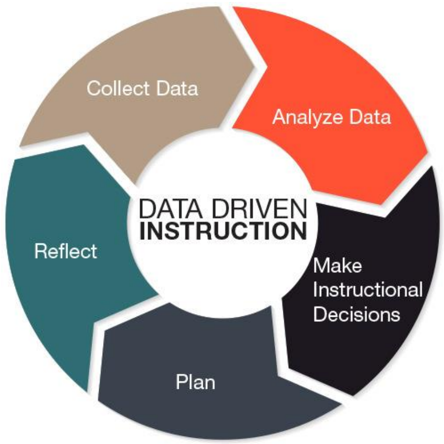
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form School: ________________ Teacher: ________________________ Date: _____________
Assessment: ____ NJSLS:
____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students <u>should retain ALL of their current artifacts</u> in their Mathematics Portfolio.

Resources

Number Book Assessment Link: **http://investigations.terc.edu/**

Model Curriculum- http://www.nj.gov/education/modelcurriculum/

Georgia Department of Education: Games to be played at centers with a partner or small group. http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten

Engage NY: *For additional resources to be used during centers or homework.
https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf

Add/ Subtract Situation Types: Darker Shading indicates Kindergarten expectations
https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf

Math in Focus PD Videos: https://www-k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.html

Number Talk/string videos https://hcpss.instructure.com/courses/124/pages/routines:

Suggested Literature

Fish Eyes by, Lois Ehlert

Ten Little Puppies by, Elena Vazquez

Zin! Zin! Zin! A Violin! by, Lloyd Moss

My Granny Went to the Market by, Stella Blackstone and Christopher Corr

Anno's Counting Book by, Mitsumasa Anno

Chicka, Chicka, 1,2,3 by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

How Dinosaurs Count to 10 by Jane Yolen and Mark Teague

10 Little Rubber Ducks by Eric Carle

Ten Black Dots by Donald Crews

Mouse Count by Ellen Stoll Walsh

Count! by Denise Fleming

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>